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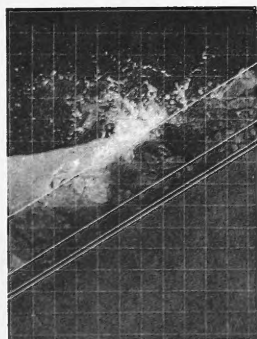
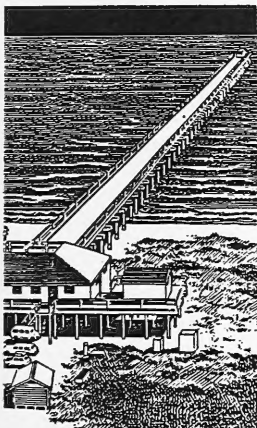
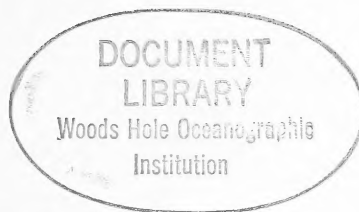
# APPROXIMATE UPPER LIMIT OF IRREGULAR WAVE RUNUP ON RIPRAP

by

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## PREFACE

Summarized herein are important findings obtained from unpublished model studies conducted by the Coastal Engineering Research Center (CERC) for the US Army Engineer District, Detroit (NCE), and the US Army Engineer District, Jacksonville (SAJ).

This report was prepared at the US Army Engineer Waterways Experiment Station (WES). The data were compiled and analyzed by Mr. John P. Ahrens, Oceanographer, and Ms. Martha S. Heimbaugh, Civil Engineer, both of CERC. The data were collected by Messrs. Martin Titus and Louis Meyerly and Ms. Karen Zirkel, Civil Engineering Technicians, CERC.

General supervision was provided by Dr. James R. Houston, Chief, CERC, Mr. Charles C. Calhoun, Jr., Assistant Chief, CERC, Dr. Charles L. Vincent, Program Manager, CERC, and Mr. C. E. Chatham, Jr., Chief, Wave Dynamics Division, CERC.

COL Dwayne G. Lee, CE, was Commander and Director of WES during the preparation and publication of this report. Dr. Robert W. Whalin was Technical Director.

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PART I: INTRODUCTION

1. In many locations riprap is the preferred type of shore protection against wave attack. The two reasons for this are the low cost and high durability of stone, and the effectiveness of randomly placed stone, because of its roughness and porosity, in dissipating wave energy and attenuating runup. Because of these reasons, riprap has been the most studied type of revetment, and its performance is well documented.

2. Runup is one of the most important factors affecting the design of revetments exposed to wave action. Generally, riprap revetments are designed so that little or no runup exceeds the top of the protection. Because of the inherent complexity of natural wave trains and the interaction of incident waves and the return flow of previous runup on a rough, porous slope, it is difficult to predict the upper limit of wave uprush on riprap. This report summarizes the most important results from two unpublished studies, and presents formulas to calculate the approximate limit of wave runup. Both studies included laboratory tests of riprap exposed to irregular wave action. The formulas can be used to compute the elevation to which protection needs to be extended to prevent exceedance by runup or to estimate the potential severity of wave overtopping.

## PART II: SOURCES OF DATA, TEST SETUPS, AND TEST CONDITIONS

3. The sources of data for this report came from model studies conducted primarily for two US Army Corps of Engineer Districts, Detroit (NCE) and Jacksonville (SAJ).

4. Model studies conducted for NCE were to investigate wave runup on riprap-protected dredge disposal dikes in the Great Lakes. The scope of this study was expanded to include an unusually wide range of water depths at the toe of the structure  $d_s$ , zero-moment wave heights  $H_{mo}$ , and period of peak energy density of the incident wave spectrum  $T_p$ . By expanding the scope of this study beyond the immediate problems occurring on the Great Lakes, the opportunity to develop a general wave-runup prediction method was provided. A summary of test conditions for both the NCE and SAJ studies is given in Table 1, and data collected on both studies are tabulated in Appendix A.

Table 1  
Summary of Test Conditions

| Study | Embankment<br>Slope | $d_s$<br>cm | $H_{mo}$<br>cm | $T_p$<br>cm | $W_{50}$<br>g | Armor<br>Unit<br>Weight<br>$g/cm^3$ | Number<br>of<br>Tests |
|-------|---------------------|-------------|----------------|-------------|---------------|-------------------------------------|-----------------------|
| NCE   | 1 on 2              | 11.9-38.5   | 4.9-17.5       | 1.02-4.74   | 189           | 2.65                                | 40                    |
| SAJ   | 1 on 3              | 19.0-23.8   | 3.0-10.5       | 1.39-1.46   | 63.3-67.0     | 2.55                                | 21                    |
| SAJ   | 1 on 4              | 19.0-23.8   | 3.2-10.3       | 1.39-1.49   | 56.9          | 2.55                                | 8                     |

5. A 1- on 2- (1 vertical:2 horizontal) structural slope was used in the NCE study. Plywood roughened with glued on pea gravel was used as the supporting slope and simulated the impermeable core of the dike. This slope was covered with a filter layer of Sioux Quartzite 5.5 cm thick. The range of weight for this stone was from 6 to 41 g with a median weight of 18 g. Riprap armor stone was placed by hand on top of the filter layer. Armor stones were composed of Kimmswick Limestone with a range of weight from 144 to 233 g with a median weight of 189 g. The armor layer was 10 cm thick. Figure 1 shows a profile view of the model structure, and Figure 2 shows a plan view of the test setup.

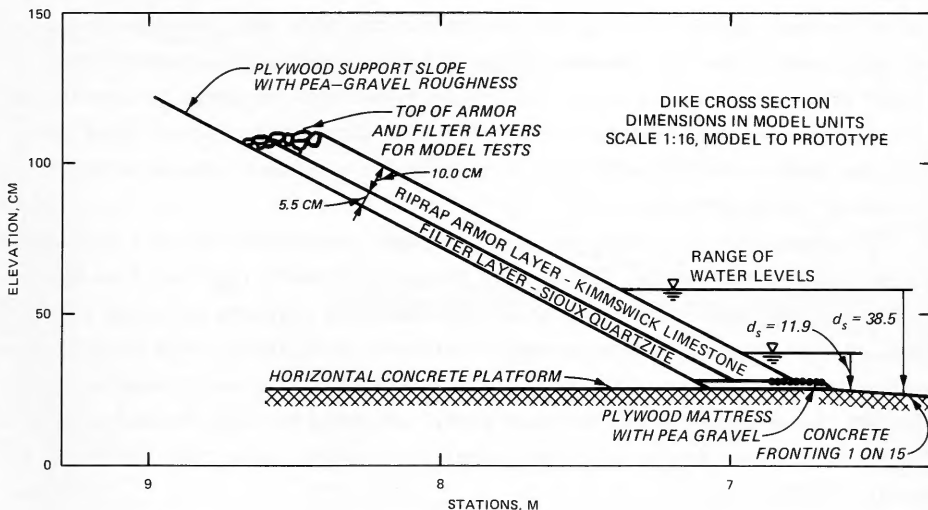


Figure 1. Profile view of structure tested during NCE study

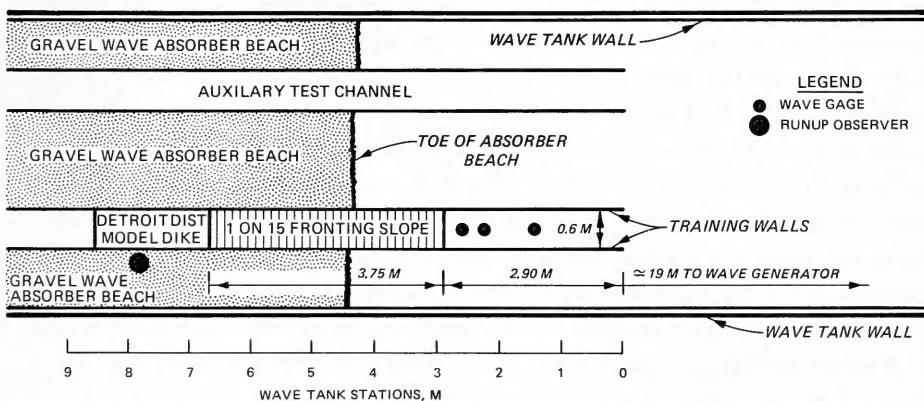


Figure 2. Plan view of test setup for NCE study

6. The model was constructed at a nominal 1:16 (model:prototype) undistorted Froude scale. The influence of scale effects at this large a scale was considered to be small. To further reduce the possibility of scale effects, the stone used in the filter layer was double the size required for geometrical similitude. Use of somewhat larger filter stones helps establish the proper flow-regime in the model filter layer when the revetment is exposed to wave action; see Broderick and Ahrens (1982). Large-sized filter stone and a 1:16 scale were used in both the NCE and the SAJ studies to minimize the influence of scale effects.

7. Tests for this study and the SAJ study were conducted in a 61-cm-wide channel within the Coastal Engineering Research Center's (CERC's) 1.2- by 4.6- by 42.7-m wave tank. Wave conditions were measured offshore by using three parallel wire-resistance wave gages. Incident and reflected wave spectra were resolved using the method of Goda and Suzuki (1976). Figure 2 shows a plan view of the wave tank setup for this study. Details relating to spectral wave generation and the analysis of wave conditions in this wave tank are given by Seelig (1980).

8. Maximum wave runup elevations were obtained by visual observations made by an experienced observer, and quantified by using a point gage. The observer stood immediately adjacent to the structure in a wave absorber channel as shown in Figure 2. The duration of the runup observation was 256 sec, corresponding to the data acquisition system's sampling interval for the wave gages to obtain the wave information. The observer tried to measure the extreme excursion of "green" water near the middle of the structure. Observations were not intended to measure the upper limit of spray or splash. Prior to using visual observations, some effort had been expended in trying to use various types of continuous wave gages positioned just above the armor surface, but runup elevations that were measured by the wave gages proved to be unreliable. After some initial observations and discussion, two experienced observers could obtain maximum runup elevations to within about a difference of 3 percent or less of each other. Additional information about the NCE study is given in Ahrens and Seelig (1980).

9. The SAJ study was conducted to investigate the stability of and wave runup on riprap to be used to protect Herbert Hoover Dike on Lake Okeechobee, Florida. Two structural slopes were tested during this study, 1 on 4 and 1 on 3. Figure 3 shows a profile view of the 1-on-4 slope tested. Figure 4 shows

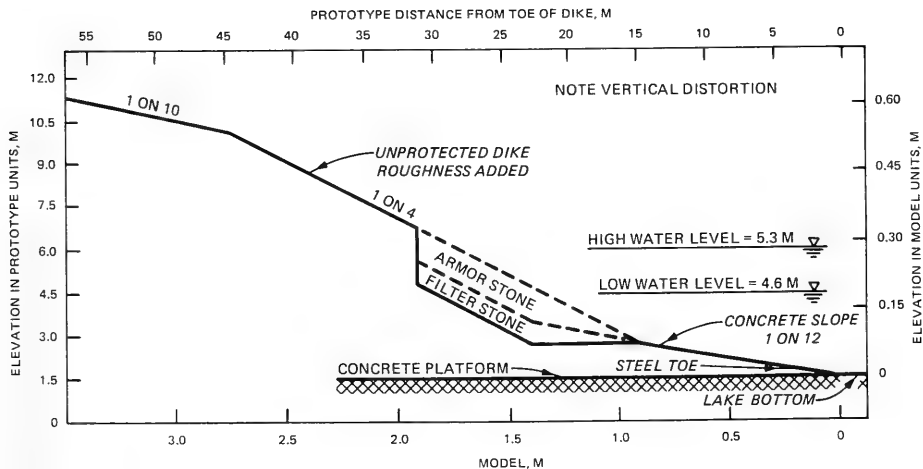


Figure 3. Profile view of the 1 on 4 structure tested during SAJ study

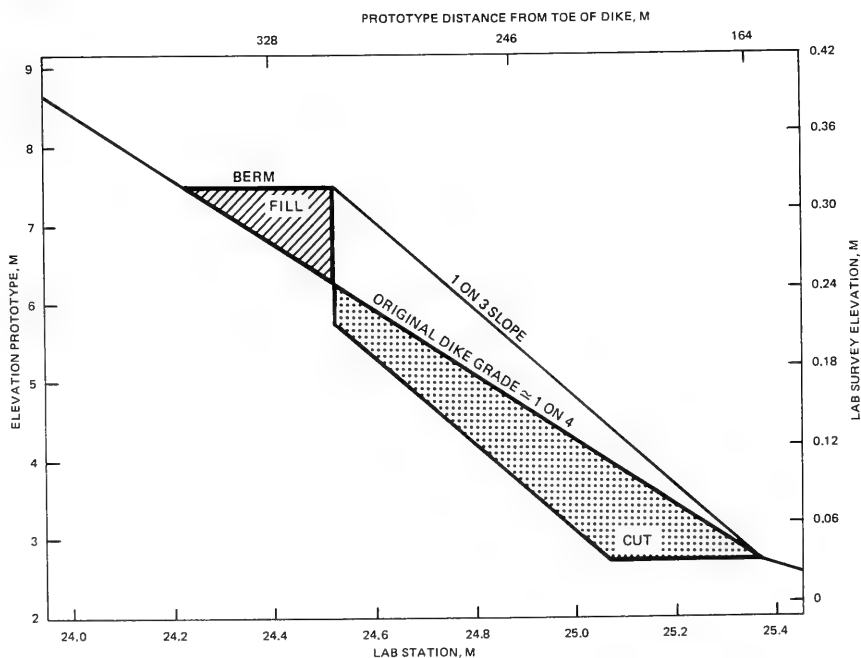


Figure 4. Profile view of cut and fill used to construct a 1 on 3 dike embankment slope for SAJ study

the cut and fill strategy used to construct a 1-on-3, riprap-protected slope on the embankment. Figure 5 shows a profile view of the 1-on-3 slope tested and the location of the wave gages. Figure 6 shows a plan view of the test setup.

10. Since the armor stone planned to protect the dike was marine limestone to be quarried in Florida, this type of stone was used in the model tests. This stone has a density of  $2.55 \text{ g/cm}^3$ . The armor stone had a median weight which ranged from about 57 to 67 g during the course of the study (see Table 1). Filter stone had a median weight of about 12 g and a layer thickness of 2.5 cm. Additional details relating to test procedures and setup are given in Ahrens and Zirkle (1982).

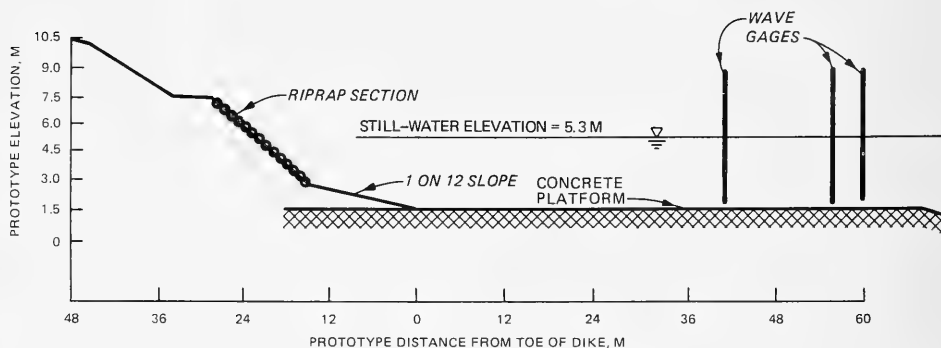


Figure 5. Profile view of 1-on-3, riprap-protected slope and offshore wave gages

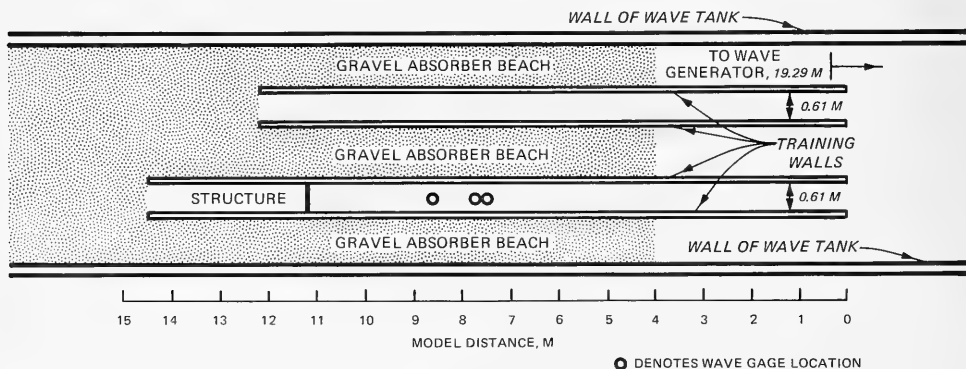


Figure 6. Plan view of SAJ test setup

PART III: ANALYSIS OF DATA AND DEVELOPMENT  
OF RUNUP FORMULAS

11. The biggest difficulty with analyzing the data from the NCE study was making accurate estimates of the zero-moment wave height  $H_{mo}$  at the toe of the structure. In the NCE study the wave heights were measured offshore in a water depth 25 cm greater than at the toe of the structure. Due to shoaling and breaking, a wide range of offshore wave conditions can yield the same zero-moment wave height in shallow water. Therefore, the offshore wave height is not as useful as the wave height at the toe of the riprap structure. The wave conditions near the structure correlate well with the runup and often can be estimated accurately by depth-limited considerations. Originally in the NCE study the wave heights at the toe were estimated by using the method of Goda (1975) which accounts for shoaling and breaking of irregular waves. However, after scrutinizing the information generated by Goda's model, it was observed that for some situations the method yielded values of  $H_{mo}/d_s$  greater than 0.8 which is higher than has been observed in any of CERC's wave tank calibration tests. Because of this limitation, it was decided to try and develop another method to estimate  $H_{mo}$  at the toe of the structure.

12. Several methods were tested to account for the wave shoaling and breaking between the offshore gages and the toe of the structure. The method that worked best was a hybrid method which combined linear-wave shoaling with the relation given by Hughes (1984) as

$$\left[ \frac{H_{mo}}{(L_p)^{3/4}} \right]_I = \left[ \frac{H_{mo}}{(L_p)^{3/4}} \right]_0 \quad (1)$$

where

$L_p$  = Airy wave length calculated at those depths for the period of peak energy density  $T_p$

I and 0 = inshore and offshore water depths, respectively

From wave-tank calibration tests it has been found that the approximate limiting value for the zero-moment wave height is given by

$$\left( \frac{H_{mo}}{L_p} \right)_{\max} = 0.10 \tanh \left( \frac{2\pi d_s}{L_p} \right) \quad (2)$$

where  $d_s$  is the water depth at or near the structure toe. The procedure used to calculate the zero-moment wave height at the toe of the structure was to calculate the value by using both linear shoaling and Equation 1 and then taking the average of the two estimates. If the average exceeded the maximum value suggested by Equation 2, then that limiting value was used.

13. The ability of the above procedure to estimate  $H_{mo}$  in shallow water is demonstrated in Figure 7 using wave-tank calibration data collected

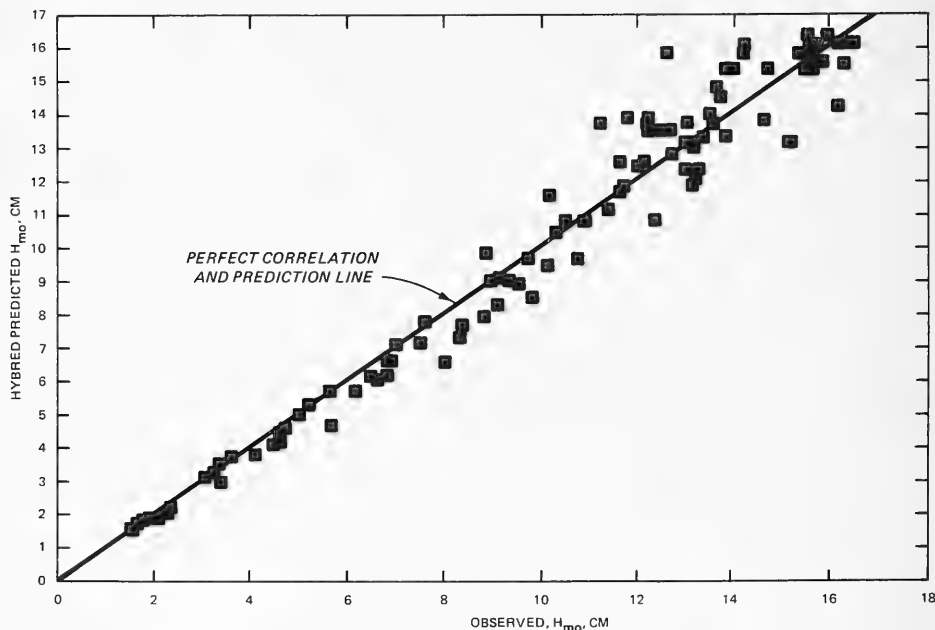


Figure 7. Predicted versus observed  $H_{mo}$  for tank calibration data prior to a study of wave overtopping of a seawall (Ahrens, Heimbaugh, and Davidson 1986). This calibration data included a wide range of wave periods and an extensive amount of wave shoaling and breaking for many conditions between the offshore wave gages and an inshore gage located in front of a wave absorber beach. For the calibration data shown in Figure 7, the offshore water depth ranged from 61.9 to 66.2 cm; the inshore water depth ranged from 22.9 to 27.2 cm; the offshore  $H_{mo}$  ranged from 1.6 to 21.5 cm; the inshore  $H_{mo}$  ranged from 1.5 to 16.4 cm; and the period of peak energy density ranged from 1.75 to 3.00 sec. The hybrid method given above appears to work well for CERC calibration data because linear shoaling tends to overestimate inshore

$H_{mo}$  while Equation 1 tends to underestimate inshore  $H_{mo}$ , and Equation 2 provides a logical limiting value on  $H_{mo}$ . Based on the success of the hybrid model in predicting known data, the model was applied to the NCE study to estimate  $H_{mo}$  at the toe of the structure. Subsequent analysis of predicted and observed maximum runup elevations suggests that the hybrid method makes good estimates of  $H_{mo}$  in shallow water.

14. The wide range of water depths tested in the NCE study had been included partly to investigate the influence of water depth on wave runup. This concern is strongly reflected in the discussion of wave runup in the Shore Protection Manual (1984). From previous studies it was known that runup would be strongly influenced by the surf condition on the structures (Ahrens and McCartney 1975), but it also seemed logical that the maximum runup would be dependent on the shape of the wave-height distribution and nonlinear effects. The last two influences would be very dependent on the water depth at the toe of the structure and the wave periods. To investigate the influence of surf characteristics on runup, the surf parameter for irregular waves  $\xi$  is defined as

$$\xi = \frac{\tan \theta}{\left(\frac{H_{mo}}{L_o}\right)^{1/2}} \quad (3)$$

where

$\tan \theta$  = tangent of the angle  $\theta$  between the structure slope and the horizontal

$L_o = gT_p^2/2\pi$  = the deep-water wave length

$g$  = the acceleration of gravity

When a runup model was formulated using the surf parameter, it was found to contain some systematic errors which could be related to the relative wave height  $H_{mo}/d_s$ . However, when a surf parameter was defined by using the local wave length, a model could be formulated which did not include systematic errors related to the relative wave height. The modified surf parameter  $\xi_L$  is defined

$$\xi_L = \frac{\tan \theta}{\left(\frac{H_{mo}}{L_p}\right)^{1/2}} \quad (4)$$

where

$L_p$  = the Airy wave length calculated by using the water depth at the toe of the structure

$d_s$  = the period of peak energy density,  $T_p$

Runup is computed using the formula

$$\frac{R_{\max}}{H_{mo}} = \frac{aS}{1.0 + bS} \quad (5)$$

where

$R_{\max}$  = elevation of maximum wave runup

$S$  = surf parameter defined by either Equation 3 or 4 depending on the prediction method selected

$a$  and  $b$  = dimensionless runup coefficients determined by regression analysis

Equation 5 has a form which is especially convenient and logical for predicting wave runup on rough porous slopes as shown by Ahrens and McCartney (1975), and, subsequently, by Seelig (1980) and US Army Corps of Engineers (1985). For the NCE data, using the modified surf parameter defined by Equation 4, the coefficients in Equation 5 were found to be  $a = 1.062$  and  $b = 0.153$ .

15. By using the coefficients given above, Equation 5 does a good job of predicting  $R_{\max}$  for both the NCE and SAJ studies. Figure 8 shows the predicted values of  $R_{\max}$  versus the observed values of  $R_{\max}$  by using different symbols to identify the two studies. The good fit to the NCE data is gratifying considering the problem related to estimating the  $H_{mo}$  at the toe of the structure. The good fit to the SAJ data is somewhat surprising considering that the thickness of the armor layer for the SAJ test was considerably thinner than the armor layer used in the NCE tests. In addition, the structural slopes tested for SAJ were 1 on 3 and 1 on 4 compared with a slope of 1 on 2 for the NCE tests. These findings indicate that the maximum runup may not be too sensitive to the armor-layer thickness, and that the surf parameter properly accounts for differences in the structural slopes. It should also be recalled that the runup coefficients were obtained from the NCE study so that the SAJ data provide a rather severe test for the runup model's predictive ability.

16. By lumping the data from the two studies together, somewhat better

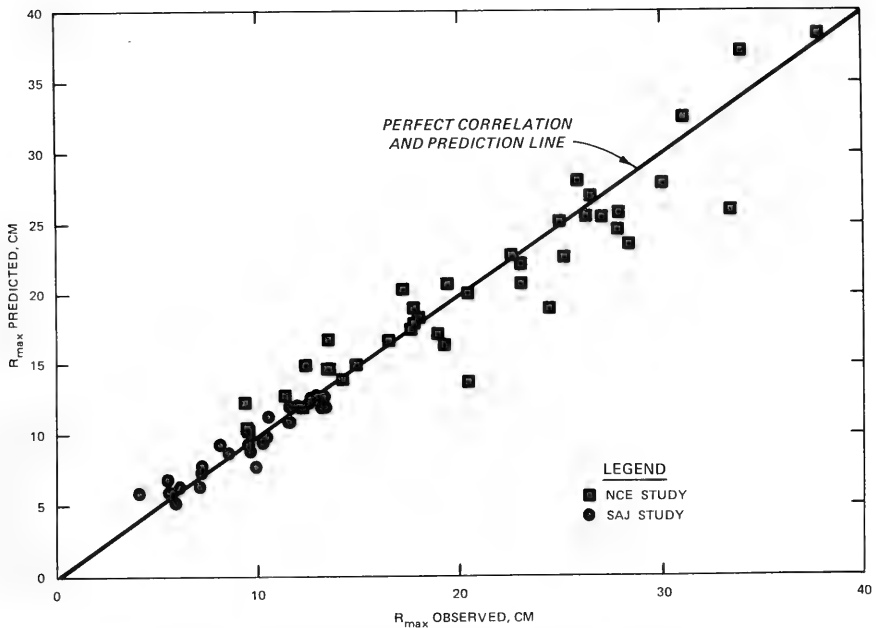


Figure 8.  $R_{max}$  predicted, using coefficients from NCE data, versus  $R_{max}$  observed, both NCE and SAJ studies

a and b runup coefficients can be determined. By using regression analysis on the combined data set, the improved coefficients are  $a = 1.154$  and  $b = 0.202$ . A new scatter plot comparing predicted and observed values was prepared with the above coefficients, and Equation 5 was used to calculate the predicted values of maximum runup. The new scatter plot (see Figure 9) shows that the change in the runup coefficients caused very little change over the scatter plot shown in Figure 8. Even though there was little change in the scatter plot, the limiting value for  $R/H_{mo}$  dropped from 6.9 to 5.7; the limiting value for Equation 5 is given by the ratio of a to b.

17. To investigate systematic error in predicting the maximum runup and to identify possible ways to improve the prediction method based on the modified surf parameter  $\xi_L$ , a series of error plots was made. In these plots, the percent error %E in predicting the maximum runup is defined as

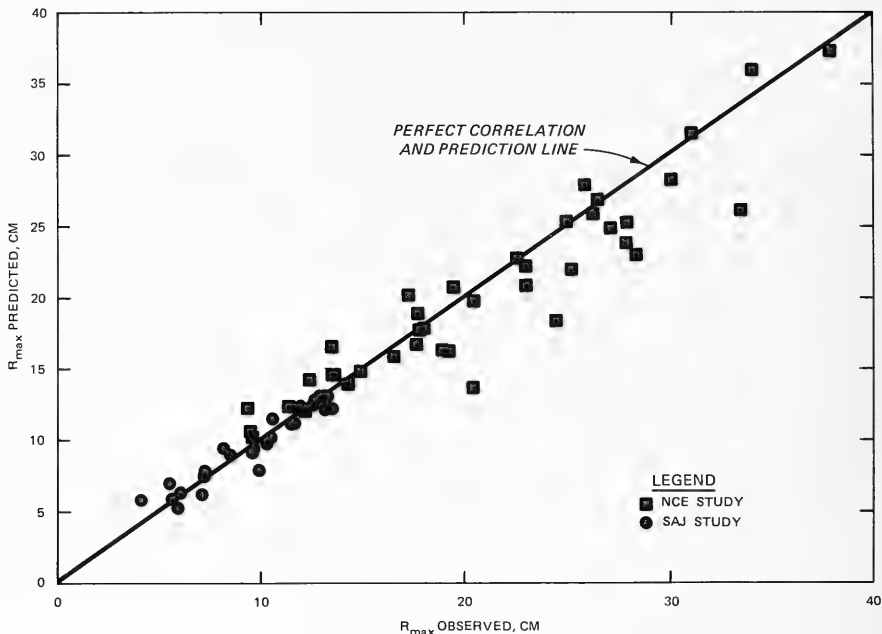


Figure 9.  $R_{\max}$  predicted, using coefficients from both NCE and SAJ data, versus  $R_{\max}$  observed for NCE and SAJ data

$$\%E = \frac{(R_{\max})_p - (R_{\max})_o}{R_{\max}_o} \times 100 \quad (6)$$

where the subscripts  $p$  and  $o$  indicate predicted and observed, respectively. The percent error is plotted versus  $\xi_L$ ,  $\xi$ ,  $d_s/L_p$ ,  $H_{mo}/L_p$ ,  $H_{mo}/d_s$ ,  $r(\text{bar})/d_{50}$ , and  $II$ , and  $\cot \theta$  in Figure 10a, b, c, d, e, f, g, and h, respectively, where  $r$  is the average armor-layer thickness and  $II$  is Goda's (1983) nonlinear parameter defined for irregular waves

$$II = \frac{\frac{H_{mo}}{L_p}}{\tan h^3\left(\frac{2\pi d_s}{L_p}\right)} \quad (7)$$

The larger the value of  $II$ , the more nonlinear the waves with

$$II \rightarrow \frac{H_{mo}}{L_p}, \text{ for deep-water conditions}$$

and

$$II \rightarrow \frac{H_{mo} L_p^2}{(2\pi d_s)^3}, \text{ Ursell's parameter for shallow-water wave conditions}$$

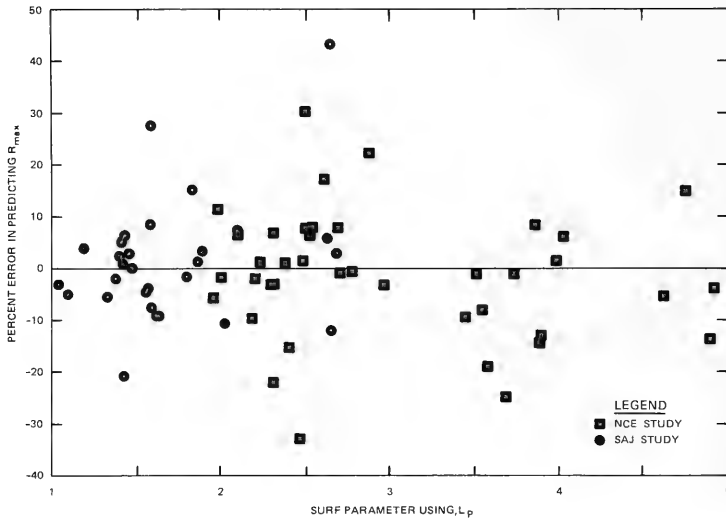
Figure 10 shows little or no systematic error for the prediction method based on  $L_p$ . Figure 10 also shows that the percent error ranges from -33 to +44 percent, but that for most tests the error is within about  $\pm 10$  percent.

18. Approximately 25 percent of the tests had a percent error greater than  $\pm 10$  percent. Because of this, it may be useful in some critical or life-threatening situations to use a value of  $R_{max}$  greater than the expected value produced by Equation 5 when using the recommended coefficients. Figure 11 shows how the percent error which has been normalized by the standard deviation of the data set  $\sigma$  seems to have the shape of a normal distribution. To test this hypothesis, namely, that the percent error has a normal distribution, a Kolmogorov-Smirnov (K&S) test was performed. This test is used to determine whether or not the data deviate a statistically significant amount from the assumed normal distribution model (Cornell and Benjamin 1970).

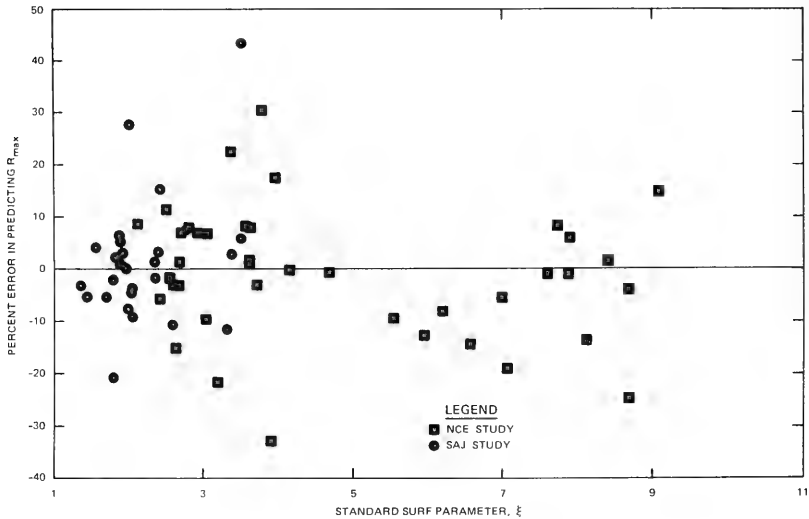
19. The K&S test indicates that the normal distribution for error should be accepted at the 20-percent significance level. A 20-percent level is a more severe criterion than a 10-percent level as it indicates there is a 20-percent chance of rejecting a model which is in fact true, a Type I error. The 20-percent significance level is the most severe criterion commonly tabulated for the K&S tests. Recognizing that errors have a normal distribution provides an easy way to give more conservative estimates of  $R_{max}$  than is provided by a regression equation. Generally, about half the errors are above the regression curve and about half are below, so the curve represents a 50 percent exceedance level. In Figure 12 a more conservative trend is shown above the regression curve. The conservative curve was generated by increasing the runup regression coefficient  $a$  by two standard deviations of the percent error, i.e.,

$$a_c \text{ (conservative } a) = a (1 + 2\sigma) = 1.143 (1.0 + 2 \times 0.1286) = 1.437$$

The value of the runup coefficient  $b$  remains the same, i.e.,  $b = 0.202$ .

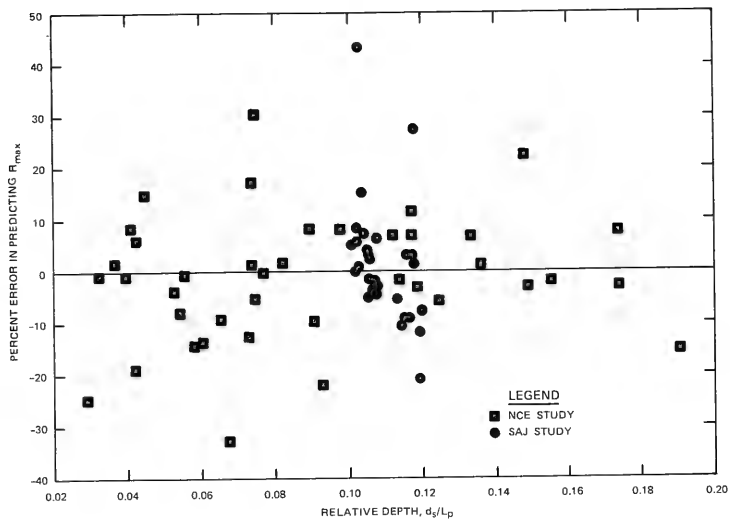


a. Percent error  $R_{max}$  versus surf parameter using  $L_p$

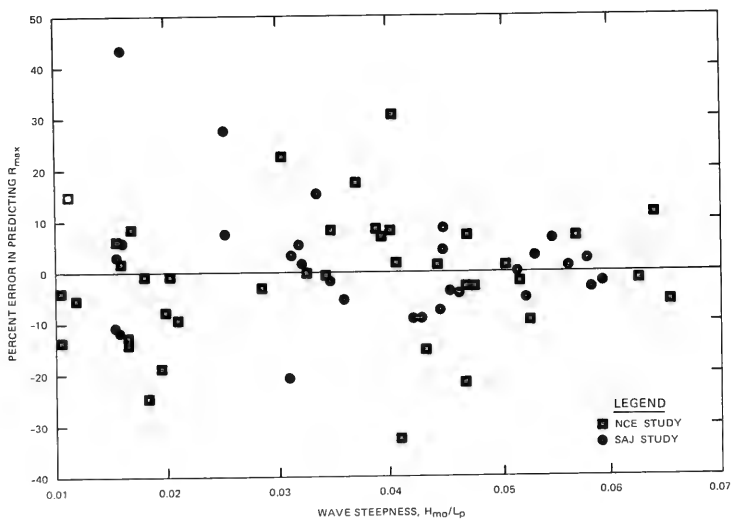


b. Percent error  $R_{max}$  versus standard surf parameter,  $\xi$

Figure 10. Percent error in predicting  $R_{max}$  (Sheet 1 of 4)

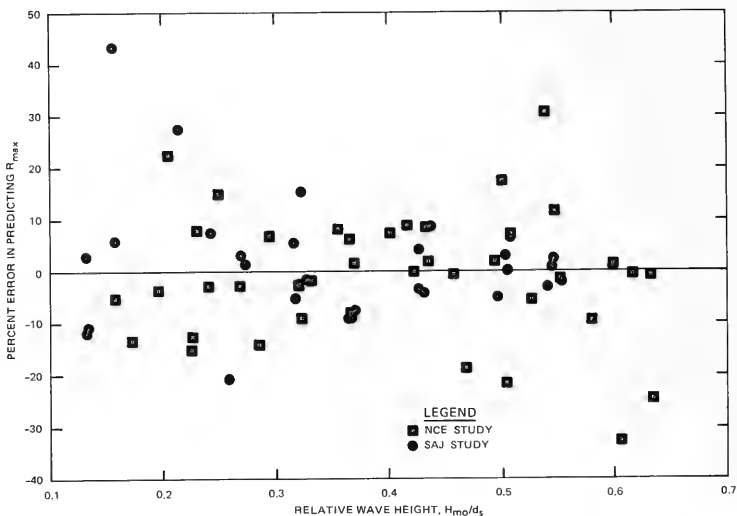


c. Percent error  $R_{max}$  versus relative depth,  $d_s/L_p$

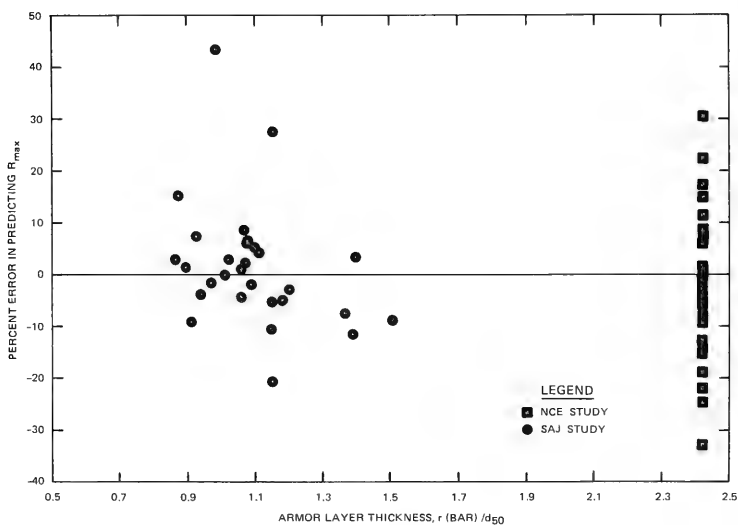


d. Percent error  $R_{max}$  versus steepness,  $H_{mo}/L_p$

Figure 10. (Sheet 2 of 4)

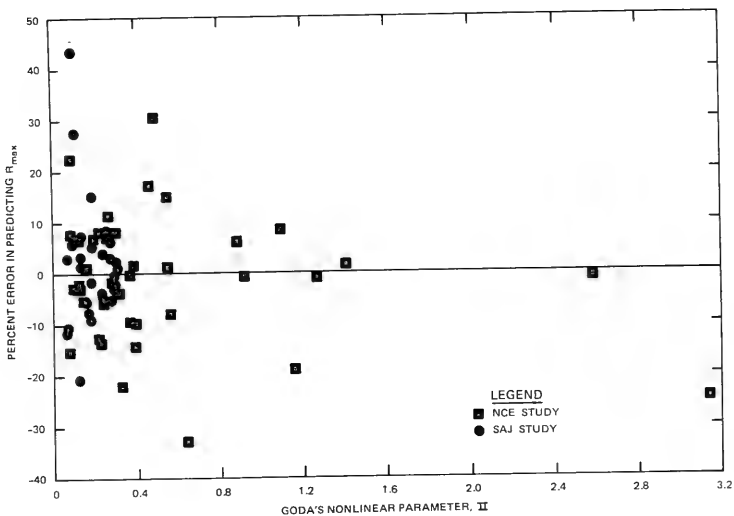


e. Percent error  $R_{max}$  versus wave height,  $H_{mo}/d_s$

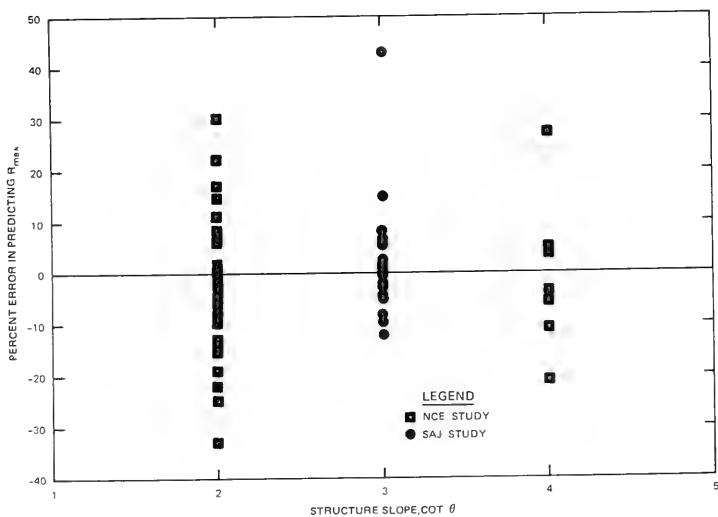


f. Percent error  $R_{max}$  versus armor-layer thickness,  $r(\bar{r})/d_{50}$

Figure 10. (Sheet 3 of 4)



g. Percent error  $R_{max}$  versus Goda's nonlinear parameter,  $II$



h. Percent error  $R_{max}$  versus structure slope,  $\cot \theta$

Figure 10. (Sheet 4 of 4)

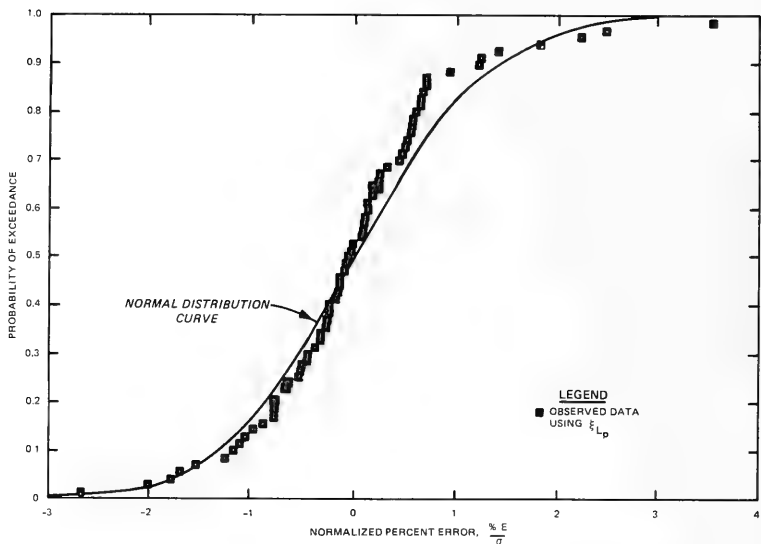


Figure 11. Normal distribution curve for  $\xi_{Lp}$  model data

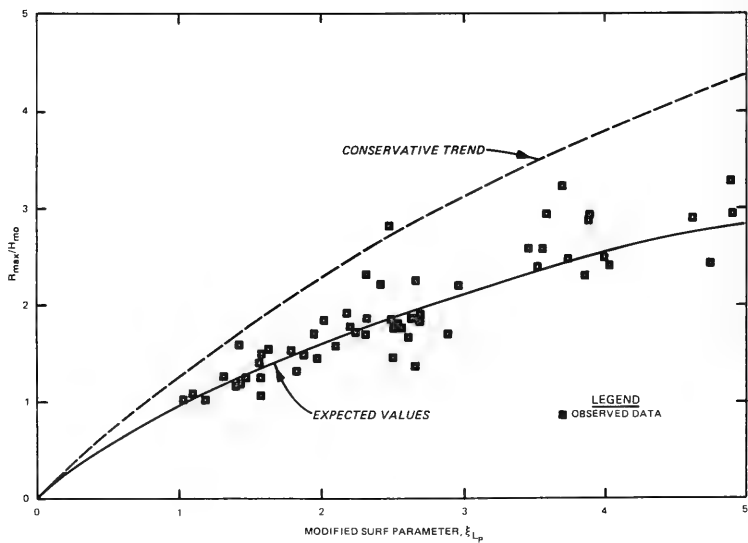


Figure 12. Expected value and more conservative trend curves for  $\xi_{Lp}$  model

It can be seen in Figure 12 that the new conservative runup curve provides an envelope for the observed data. The conservative curve would be expected to exceed about 97.7 percent of the data. An exceedance level of 97.7 percent is obtained from a standard normal distribution table for a value two standard deviations greater than the mean. Figure 12 helps confirm the method of choosing an envelope curve by showing only one observed value above the conservative curve. This is approximately what would be expected for a normal distribution with a sample size of 69. Other curves used to predict maximum runup could be constructed which would be more or less conservative than the example just provided. The degree of conservatism would be evaluated on the basis of the risk posed by waves overtopping the revetment.

20. Since the standard surf parameter has been frequently used to predict wave runup, it is useful to provide a prediction formula based on that method to allow comparison to earlier studies. Using Equation 3 to define the surf parameter in Equation 5, the runup coefficients were determined for the combined NCE and SAJ data sets as  $a = 1.022$  and  $b = 0.247$ . Figure 13 shows the predicted and observed values of  $R_{\max}/H_{mo}$  versus the standard surf parameter. It can be seen that the predicted values follow the trend of the observed data very well. Using the same method to evaluate errors as was used

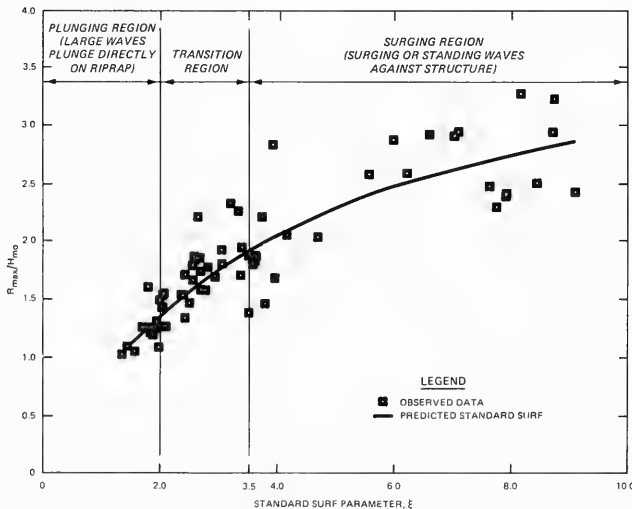


Figure 13. Comparison of the prediction of  $R_{\max}$  using  $\xi$

for the Equation 4 model, it was found that there were no systematic errors associated with the model which used the standard surf parameter. Figure 14 shows that the standardized percent errors for this model also seem to have a normal distribution. Performing the K&S test once again showed that these data were also normal at the 20-percent significance level and thus, could be assumed to have a normal distribution. Figure 15 shows the more conservative curve which could be expected to envelop 97.7 percent of the data and represents an increase of two standard deviations over the expected mean curve. The coefficients for this conservative curve are  $a = 1.285$  and  $b = 0.247$ . Once again, this curve is only one of many more conservative curves that could be constructed depending upon the design situation. Using the runoff coefficients with the standard surf parameter would be an easy way to estimate  $R_{\max}$  using a small calculator. The more accurate model would require the calculation of  $L_p$  for use in the modified surf parameter which would be more difficult than calculating the deep-water wave length for the standard surf parameter.

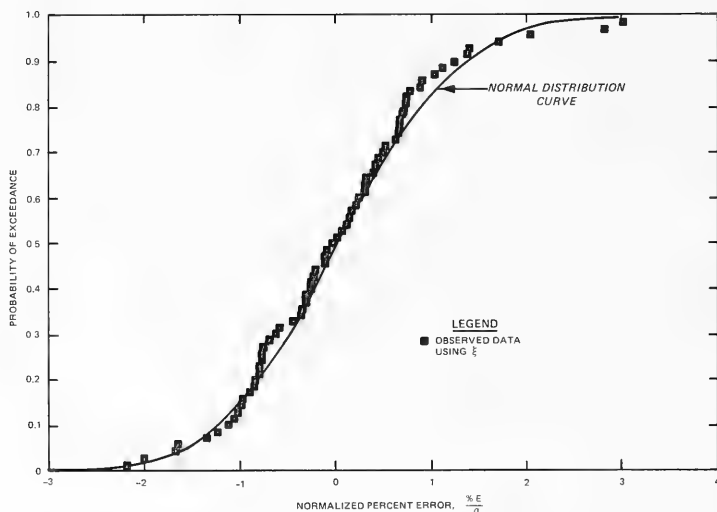


Figure 14. Normal distribution curve for  $\xi$  model data

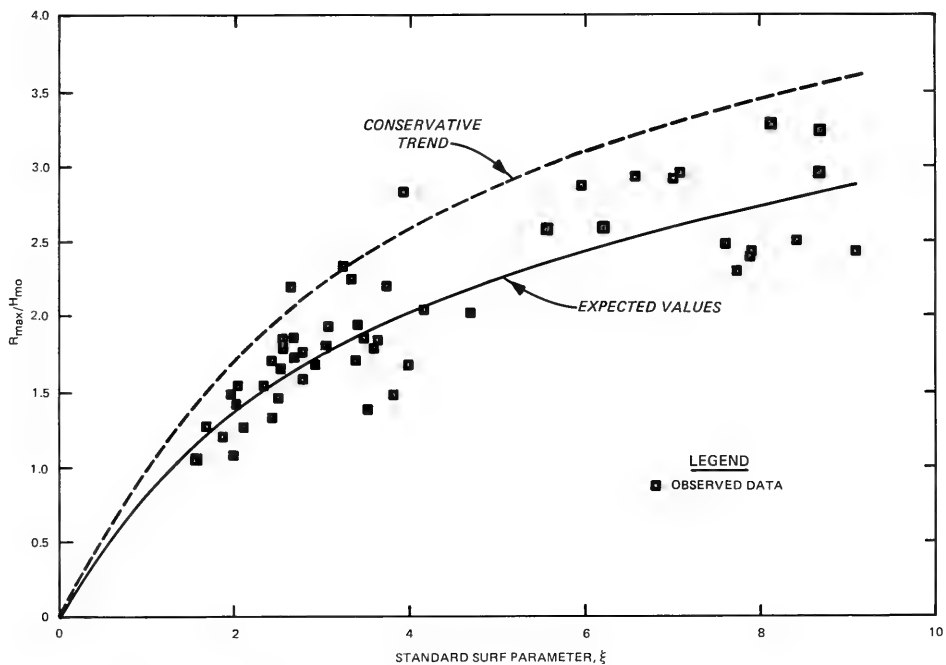


Figure 15. Expected value and more conservative trend curves for  $\xi$  model

# PART IV: SUMMARY

21. All the equations presented within this report were developed from two unpublished laboratory studies. These equations provide an easy way to calculate the  $R_{\max}$  of irregular waves on riprap-protected embankments. Table 2 summarizes the important information about two runup equations which

Table 2  
Summarized Information for Maximum Runup Formula, Equation 5

| Formula Category | Wave Length Used | Surf Parameter Used in Equation 5 | Runup Coefficients         | Variance Explained | $\sigma$ , Standard Deviation of Percent Error |
|------------------|------------------|-----------------------------------|----------------------------|--------------------|--|
| Recommended      | $L_p$            | $\xi_L$ (Equation 4)              | $a = 1.154$<br>$b = 0.202$ | $R^2 = 0.843$      | 12.3   |
| Alternative      | $L_o$            | $\xi$ (Equation 3)                | $a = 1.022$<br>$b = 0.247$ | $R^2 = 0.817$      | 12.9   |

represent the most accurate existing method to determine the approximate upper limit of wave uprush on a riprap revetment. The two equations are presented as a recommended method and an alternate method to compute  $R_{\max}$ . The recommended method has little or no systematic error such as might be associated with the influence of water depth or nonlinear effects and is slightly more accurate than the alternate method. The alternative method is easier to calculate and can serve as a "rule of thumb" estimate. In Table 2 the runup coefficients are to be used in the general runup equation (Equation 5) by using either the standard or modified surf parameter as noted. A method was developed which provides a reasonable way to make the predicted values of  $R_{\max}$  more conservative. It was found that the errors in predicting  $R_{\max}$  have a normal distribution, and this fact was used to adjust the runup coefficient  $a$  so that any predetermined exceedance level for  $R_{\max}$  could be achieved. For example, by increasing the coefficient  $a$  by two standard deviations of the percent error gives  $R_{\max}$  predictions which would be expected to exceed 97.7 percent of the observed values of  $R_{\max}$ . This technique produces a logical envelope for the data. Table 2 lists the standard deviation  $\sigma$  of

the percent error which was used in this method and the correlation squared which is the variance explained by the regression equation used to predict  $R_{\max}/H_{\text{mo}}$  .

## PART V: CONCLUSIONS AND RECOMMENDATIONS

22. Visual observations of the maximum runup of irregular waves on riprap give reliable values of the maximum elevation of wave uprush. For the two studies considered the time interval of observation was 256 sec. It is difficult for an observer to maintain adequate concentration on the runup process for intervals longer than 256 sec. This interval provides between 100 and 250 runup events at the wave periods tested. Future tests will consider using photogrametric methods to measure irregular wave runup on riprap to increase the time interval of observation and to obtain the entire runup distribution rather than just the maximum value.

23. The runup equations presented appear to be the best available to estimate the approximate upper limit of irregular wave uprush on riprap revetments. Further tests are planned which should produce improved methods to determine the runup characteristics of irregular waves on rough and porous slopes.

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## APPENDIX A

### SUMMARY OF JACKSONVILLE AND DETROIT DISTRICT TEST CONDITIONS



Table A1

## Summary of Jacksonville (SAJ) and Detroit District (NCE) Test Conditions

| study design | Test No. | offshore wave |        | offshore |        | inshore estimate |         | inshore wave |        | structure | offshore median |       | armor unit | armor layer | median weight | filter stone | typical filter | typical diamsn. | armor over d50 |
|--------------|----------|---------------|--------|----------|--------|------------------|---------|--------------|--------|-----------|-----------------|-------|------------|-------------|---------------|--------------|----------------|-----------------|----------------|
|              |          | depth         | period | Hao      | depth  | Hao              | depth   | Hao          | length |           | slope           | slope | weight     | thickness   | weight        | unit         | layer          | diamsn.         |                |
|              |          | cm            | sec    | cm       | cm     | cm               | cm      | cm           | cm     |           | theta           | alpha | kg         | cm          | kg            | cm           | cm             | cm              |                |
| Detroit      | 1        | 36,981        | 2.753  | 5.877    | 11,981 | 4,945            | 294,871 | 11,392       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 2        | 36,981        | 1.533  | 6.979    | 11,981 | 6,394            | 159,919 | 9,392        | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 3        | 36,981        | 1.820  | 7.348    | 11,981 | 6,496            | 181,784 | 9,586        | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 4        | 36,981        | 3.021  | 10,218   | 11,981 | 7,544            | 218,518 | 24,483       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 5        | 36,981        | 2.832  | 9.536    | 11,981 | 7,354            | 215,118 | 14,897       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 6        | 36,981        | 1.684  | 13,171   | 11,981 | 7,282            | 176,742 | 28,483       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 7        | 45,580        | 3.282  | 5.258    | 28,580 | 5,899            | 459,122 | 12,482       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 8        | 45,580        | 1.862  | 7,424    | 28,580 | 6,797            | 132,112 | 12,192       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 9        | 45,580        | 1.571  | 7,780    | 28,580 | 7,252            | 210,219 | 13,586       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 10       | 45,580        | 1.954  | 9,858    | 28,580 | 8,644            | 267,880 | 17,793       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 11       | 45,580        | 3.587  | 9,858    | 28,580 | 9,584            | 491,389 | 28,388       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 12       | 45,580        | 2.832  | 10,718   | 28,580 | 10,239           | 278,362 | 17,282       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 13       | 45,580        | 2.832  | 13,482   | 28,580 | 12,274           | 278,362 | 22,498       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 14       | 45,580        | 1.684  | 15,777   | 28,580 | 11,870           | 227,174 | 22,993       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 15       | 53,581        | 3.282  | 5,747    | 28,581 | 5,581            | 558,626 | 16,497       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 16       | 53,581        | 1.148  | 8,289    | 28,581 | 7,638            | 165,885 | 14,347       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 17       | 53,581        | 1.588  | 12,864   | 28,581 | 11,418           | 243,886 | 19,393       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 18       | 53,581        | 1.948  | 15,884   | 28,581 | 14,339           | 387,781 | 35,395       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 19       | 53,581        | 1.628  | 17,778   | 28,581 | 15,491           | 251,882 | 26,194       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 20       | 53,581        | 2.178  | 14,632   | 28,581 | 14,840           | 347,857 | 26,483       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 21       | 53,581        | 4.741  | 12,687   | 28,581 | 12,388           | 765,372 | 38,994       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 22       | 53,581        | 3.288  | 10,737   | 28,581 | 10,428           | 524,679 | 26,994       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 23       | 58,299        | 3.122  | 5,926    | 32,299 | 5,753            | 558,735 | 18,898       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 24       | 58,299        | 1.243  | 8,155    | 32,299 | 7,649            | 191,683 | 15,882       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 25       | 58,299        | 1.533  | 10,331   | 32,299 | 9,779            | 258,539 | 17,697       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 26       | 58,299        | 2.178  | 14,971   | 32,299 | 14,374           | 373,358 | 25,794       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 27       | 58,299        | 1.628  | 18,419   | 32,299 | 17,486           | 267,762 | 28,884       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 28       | 58,299        | 3.241  | 9,765    | 32,299 | 9,486            | 578,688 | 27,794       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 29       | 58,299        | 4.741  | 11,573   | 32,299 | 15,284           | 847,677 | 37,788       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 30       | 58,299        | 2.989  | 13,803   | 32,299 | 18,742           | 511,341 | 27,788       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 31       | 63,580        | 2.753  | 6,239    | 38,580 | 6,847            | 216,482 | 17,482       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 32       | 63,580        | 2.249  | 9,225    | 38,580 | 8,784            | 282,284 | 19,282       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 33       | 63,580        | 1.515  | 8,288    | 38,580 | 7,983            | 248,949 | 15,586       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 34       | 63,580        | 1.816  | 9,678    | 38,580 | 9,249            | 325,817 | 20,483       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 35       | 63,580        | 1.586  | 12,946   | 38,580 | 12,281           | 259,852 | 22,898       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 36       | 63,580        | 4.741  | 14,381   | 38,580 | 14,851           | 910,831 | 33,989       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 37       | 63,580        | 1.628  | 14,931   | 38,580 | 14,211           | 283,661 | 24,898       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 38       | 63,580        | 2.813  | 8,992    | 38,580 | 8,728            | 528,574 | 25,188       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 39       | 36,981        | 1.862  | 6,683    | 11,981 | 6,829            | 186,618 | 9,681        | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| Detroit      | 40       | 36,981        | 3.468  | 7,885    | 11,981 | 7,525            | 371,834 | 18,882       | 2,800  | 15.00     | 187.00          | 2.65  | 18.00      | 18.00       | 2.65          | 5.50         | 4.13           | 1.89            | 2.42           |
| SAJ          | 3        | 48,812        | 1.498  | NA       | 23,812 | 7,544            | 217,488 | 9,681        | 4,000  | NA        | 56.90           | 2.55  | 3.24       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.15           |
| SAJ          | 4        | 48,812        | 1.448  | NA       | 23,812 | 5,886            | 282,680 | 5,585        | 4,000  | NA        | 56.90           | 2.55  | 3.24       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.15           |
| SAJ          | 8        | 48,812        | 1.428  | NA       | 23,812 | 6,153            | 199,788 | 9,986        | 4,000  | NA        | 56.90           | 2.55  | 3.24       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.15           |
| SAJ          | 9        | 48,812        | 1.488  | NA       | 23,812 | 3,181            | 288,088 | 5,986        | 4,000  | NA        | 56.90           | 2.55  | 3.24       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.15           |
| SAJ          | 10       | 44,858        | 1.398  | NA       | 19,858 | 18,268           | 177,488 | 18,497       | 4,000  | NA        | 56.90           | 2.55  | 3.24       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.18           |
| SAJ          | 11       | 44,858        | 1.428  | NA       | 19,858 | 9,438            | 181,488 | 18,386       | 4,000  | NA        | 56.90           | 2.55  | 3.32       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.18           |
| SAJ          | 12       | 44,858        | 1.428  | NA       | 19,858 | 8,115            | 181,088 | 9,592        | 4,000  | NA        | 56.90           | 2.55  | 3.13       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.11           |
| SAJ          | 13       | 44,858        | 1.478  | NA       | 19,858 | 6,881            | 189,488 | 7,281        | 4,000  | NA        | 56.90           | 2.55  | 3.09       | 11.68       | 2.65          | 2.58         | 2.82           | 1.64            | 1.18           |
| SAJ          | 25       | 48,812        | 1.458  | NA       | 23,812 | 8,725            | 284,788 | 13,586       | 3,000  | NA        | 66.80           | 2.55  | 4.45       | 11.68       | 2.65          | 2.58         | 2.76           | 1.64            | 1.48           |
| SAJ          | 26       | 48,812        | 1.468  | NA       | 23,812 | 6,481            | 285,988 | 9,792        | 3,000  | NA        | 66.80           | 2.55  | 4.14       | 11.68       | 2.65          | 2.58         | 2.76           | 1.64            | 1.48           |
| SAJ          | 27       | 48,812        | 1.428  | NA       | 23,812 | 3,143            | 199,788 | 7,186        | 3,000  | NA        | 66.80           | 2.55  | 4.11       | 11.68       | 2.65          | 2.58         | 2.76           | 1.64            | 1.39           |
| SAJ          | 38       | 48,812        | 1.428  | NA       | 23,812 | 8,581            | 198,988 | 13,282       | 3,000  | NA        | 66.80           | 2.55  | 4.04       | 11.68       | 2.65          | 2.58         | 2.76           | 1.64            | 1.37           |
| SAJ          | 31       | 44,858        | 1.418  | NA       | 19,858 | 18,382           | 188,388 | 12,786       | 3,000  | NA        | 64.80           | 2.55  | 3.15       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.88           |
| SAJ          | 32       | 44,858        | 1.398  | NA       | 19,858 | 9,639            | 177,488 | 11,681       | 3,000  | NA        | 64.80           | 2.55  | 3.16       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.87           |
| SAJ          | 33       | 44,858        | 1.468  | NA       | 19,858 | 8,235            | 186,088 | 10,592       | 3,000  | NA        | 64.80           | 2.55  | 3.14       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.87           |
| SAJ          | 34       | 44,858        | 1.418  | NA       | 19,858 | 6,218            | 188,388 | 9,681        | 3,000  | NA        | 64.80           | 2.55  | 3.25       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.88           |
| SAJ          | 35       | 44,858        | 1.468  | NA       | 19,858 | 2,791            | 186,088 | 5,581        | 3,000  | NA        | 64.80           | 2.55  | 3.17       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.88           |
| SAJ          | 36       | 44,858        | 1.398  | NA       | 19,858 | 18,516           | 177,788 | 13,297       | 3,000  | NA        | 64.80           | 2.55  | 3.21       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.89           |
| SAJ          | 37       | 44,858        | 1.428  | NA       | 19,858 | 9,544            | 181,088 | 12,882       | 3,000  | NA        | 64.80           | 2.55  | 3.00       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.82           |
| SAJ          | 38       | 44,858        | 1.488  | NA       | 19,858 | 8,192            | 176,188 | 11,497       | 3,000  | NA        | 64.80           | 2.55  | 3.12       | 11.68       | 2.65          | 2.58         | 2.94           | 1.64            | 1.86           |





